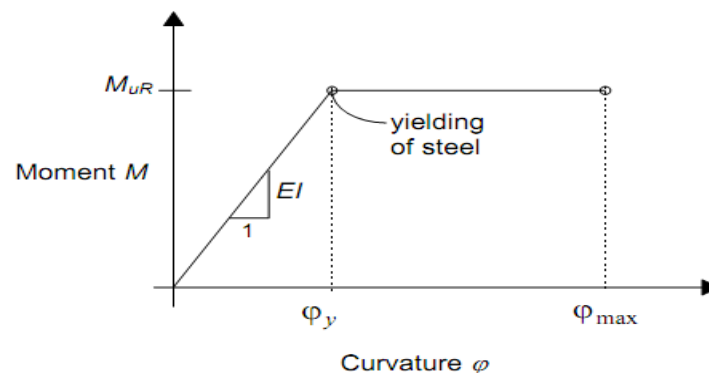


Chapter 3: Redistribution of Moments

Introduction

Statically indeterminate structures made of reinforced concrete like fixed ended one span beams, continuous beams and frames are designed considering internal forces like bending moment, shear force and axial thrust obtained from elastic structural analysis. Either one or several sections of these structures may have peak values of the internal forces, which are designated as critical sections. These sections are dimensioned and reinforced accordingly.

Flexural members, however, do not collapse immediately as soon as the loads at a particular section that have a bending moment exceeding the maximum resisting moment capacity of that section. Instead, that section starts rotating at almost constant moment. This is known as formation of plastic hinge at that section reaching its maximum resisting moment capacity. The section then transfers loads to other sections if the applied loads are further increased. This process continues till the structures have plastic hinges at sufficient sections to form a failure mechanism when it actually collapses. However, significant transfer of loads has to occur before the collapse of the structure. This assumption is approximately true for under-reinforced sections. Thus the term moment redistribution is generally used to refer to the transfer of moments to the less stressed sections as sections of peak moments yield on their ultimate capacity being reached. In other words, this transfer of loads after the formation of first plastic hinge at sections having the highest bending moment till the collapse of the structure is known as redistribution of moments. By this process, therefore, the structure continues to accommodate higher loads before it collapses.



The elastic bending moment diagram prior to the formation of first plastic hinge and the final bending moment diagram just before the collapse are far different. The ratio of the negative to positive elastic bending moments is no more valid. The development of plastic hinges depends on the available plastic moment capacity at critical sections. It is worth mentioning that the redistribution of moment is possible if the section forming the plastic hinge has the ability to rotate at constant moment, which depends on the amount of reinforcement actually provided at that section. The section must be under-reinforced and should have sufficient ductility.

This phenomenon is well known in steel structures. However, the redistribution of moment has also been confirmed in reinforced concrete structure by experimental investigations. It is also a fact that reinforced concrete structures have comparatively lower capacity to rotate than steel structures. Yet, this phenomenon is drawing the attention of the designers.

Presently, design codes of most of the countries allow the redistribution up to a maximum limit because of the following advantages:

- 1) It gives a more realistic picture of the actual load carrying capacity of the indeterminate structure.
- 2) Structures designed considering the redistribution of moment (though it is limited) would result in economy as the actual load capacity is higher than that we determine from any elastic analysis.
- 3) The designer enjoys the freedom of modifying the design bending moments within the limits. These adjustments are sometimes helpful in reducing the reinforcing bars, which are crowded, especially at locations of high bending moment.

Moment redistribution according to EBCS 2, 1995

Moments obtained from a linear analysis may be reduced by multiplying by the reduction coefficient δ provided that the moments are increased in other sections in order to maintain equilibrium.

- ❖ For continuous beams and for beams in rigid jointed braced frames with span/effective-depth ratio not greater than 20,

$$\delta \geq 0.44 + 1.25\left(\frac{x}{d}\right)$$

The neutral axis height, x , is calculated at the ultimate limit state and the term x/d refers to the section where the moment is reduced.

- ❖ For other continuous beams and rigid jointed braced frames

$$\delta \geq 0.75$$

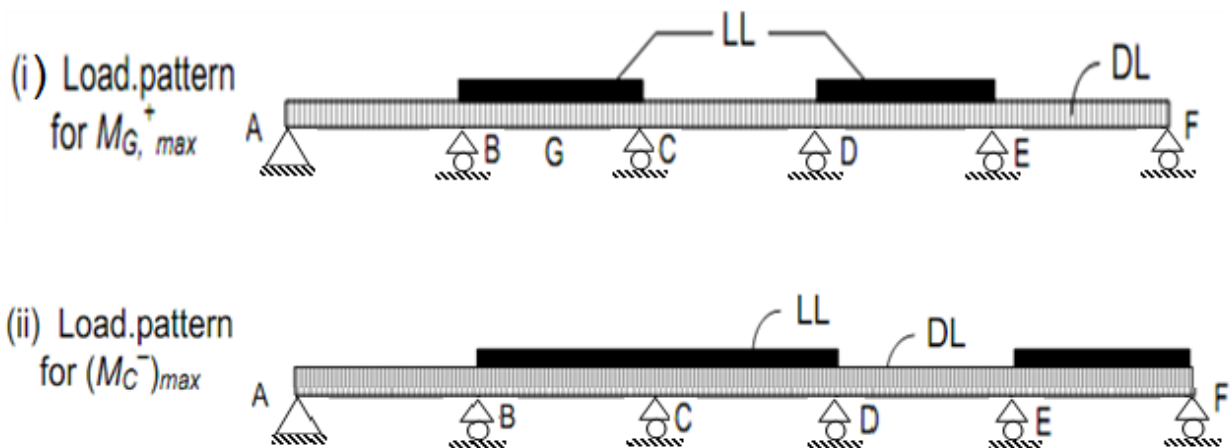
- ❖ Moment redistribution should not be allowed for indeterminate structures analyzed by inelastic analysis.

Gravity load patterns for maximum design moments

- Gravity loads comprises dead loads and live loads. Dead loads, by their inherent nature, act at all times in a fixed position, whereas live loads occur randomly both temporally (relating to measured time) and spatially (relating to measured position). In order to determine the maximum ('positive' as well as 'negative') moments that can occur at any section in a continuous beam, it is first necessary to identify the spans to be loaded with live loads so as to create the 'worst' (most extreme) effects.

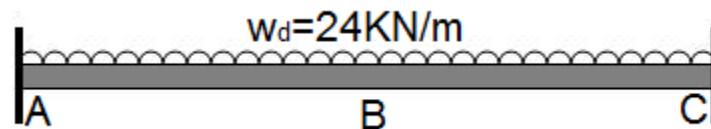
The following conclusions may be drawn regarding the positioning of live loads for obtaining the critical design moments in continuous beams:

- i. The maximum 'positive' moment in a span occurs when live loads are placed on that span and every other alternate span.
- ii. The maximum 'negative' moment at a support section occurs when live loads are placed on two adjacent spans in which the support section is located I between and also on every alternate span thereafter.



Example 1

Draw the design bending moment diagram of the beam of Fig. (a), fixed at both ends and carrying an ultimate factored uniformly distributed load of 24kN/m with a redistribution of 30 per cent.



Solution:

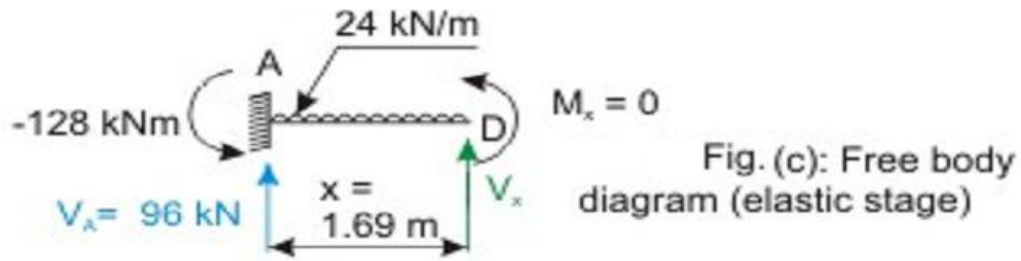
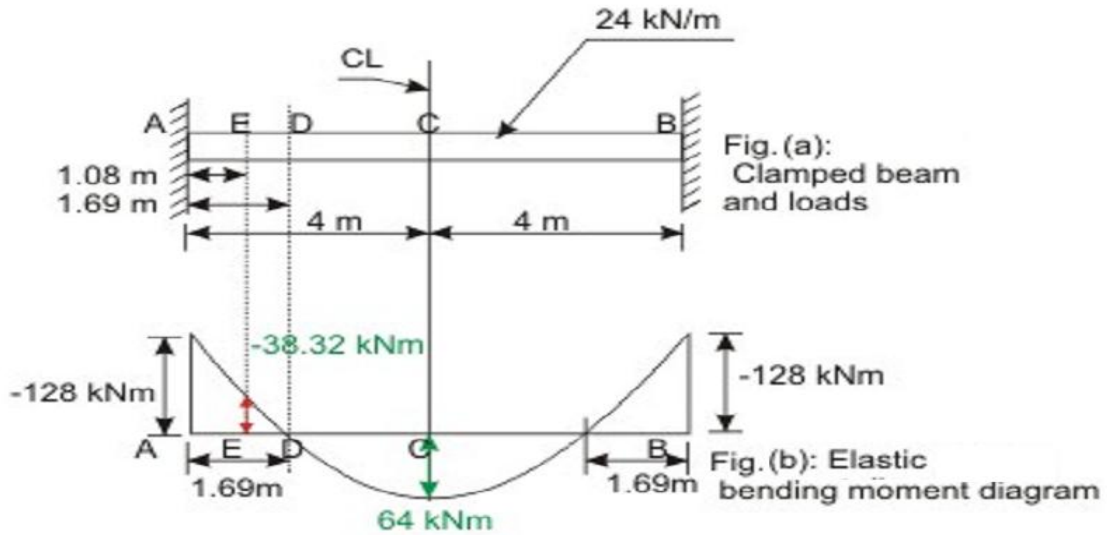
Step 1: Elastic bending moment diagram

The beam Fig. (a) is statically indeterminate. The elastic bending moment diagram is shown in Fig. (b). The negative and positive moments at the supports (A and B) and mid-span (C), respectively, are:

$$M_A = M_B = \frac{wl^2}{12} = -24 * \frac{8^2}{12} = -128 \text{KNm}$$

$$M_C = \frac{wl^2}{24} = 24 * \frac{8^2}{24} = 64 \text{KNm}$$

The distance x where the bending moment is zero is obtained by taking moments of forces of the free body diagram Fig. (c) about D after determining the vertical reaction at A.



Vertical reaction at A,

$$V_A = \frac{wl}{2} = \frac{24 * 8}{2} = 96KN$$

At section D, $M_x = 96x - 128 - 12x^2$

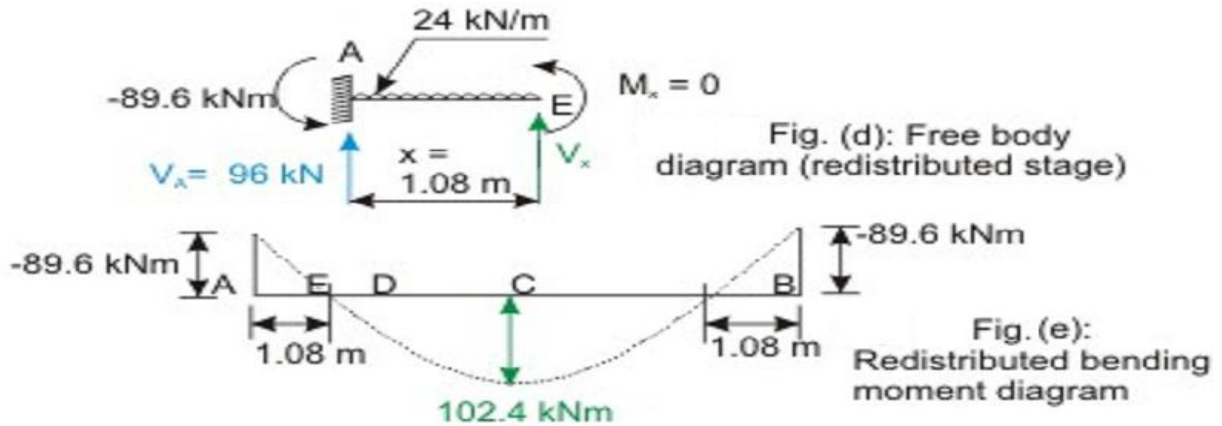
So, the value of x when $M_x = 0$ is obtained from

$$96x - 128 - 12x^2 = 0 \text{ or } 3x^2 - 24x + 32 = 0$$

The solution of the equation is $x = 1.69$ m and 6.31 m.

Step 2. Redistributed bending moment diagram

From Fig.e, the maximum negative moment at A after the full redistribution of 30 per cent = $0.7 (128) = 89.6 \text{ kNm}$.



So, the maximum positive moment at the mid-span C

$$M_c = \frac{wl^2}{8} - 89.6 = \frac{wl^2}{24} + 0.3 * \frac{wl^2}{12} = 102.4 \text{ kNm}$$

The vertical reaction at A is: $V_A = 96 \text{ kN}$.

The point of inflection, i.e., the point where bending moment is zero, is obtained by taking moment of forces about E of the free body diagram shown in Fig. d. Thus, we have:

$$96x - 12x^2 - 89.6 = 0$$

which gives $x = 1.08 \text{ m}$ and 6.92 m . The bending moment diagram after the redistribution is shown in Fig.e.

Step 3. Envelope of bending moment diagrams

At $x = 1.08 \text{ m}$, the elastic bending moment is $M_x = 96x - 12x^2 - 128 = -38.32 \text{ kNm}$, which becomes $0.7 (-38.32) = -26.824 \text{ kNm}$ after the redistribution. The envelope of the bending moment diagram is shown in Fig.f.

