DAV University Jalandhar

Department of Physics

Study Material: PHY 254 Electricity & Magnetism

B.Sc (Hons.) Physics 4th sem

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BOOK: Electricity & Magnetism by A K Sikri

Module 3 : MAGNETIC FIELD
Lecture 15 : Biot- Savarts' Law

Objectives

In this lecture you will learn the following

- Study Biot-Savart's law
- Calculate magnetic field of induction due to some simple current configurations.
- Define magnetic moment of a current loop.
- Find force between two current carrying conductors.

Biot- Savarts' Law

Biot-Savarts' law provides an expression for the magnetic field due to a current segment. The field $d\vec{B}$ at a position $\vec{r}$ due to a current segment $I\vec{dl}$ is experimentally found to be perpendicular to $\vec{dl}$ and $\vec{r}$. The magnitude of the field is proportional to the length $|dl|$ and to the current I and to the sine of the angle between $\vec{r}$ and $\vec{dl}$.

inversely proportional to the square of the distance r of the point P from the current element.
Mathematically,

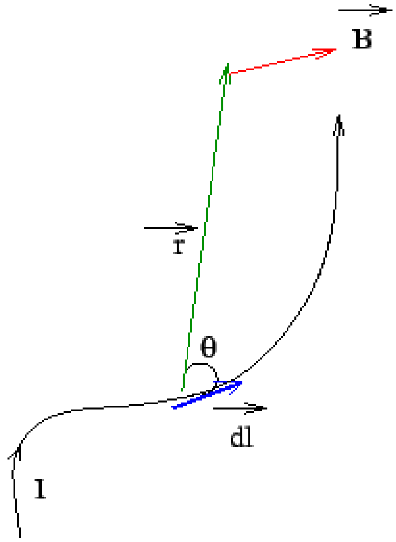
$$d\vec{B} \propto I\frac{\vec{dl}\times\hat{r}}{r^2}$$

In SI units the constant of proportionality is $\mu_0/4\pi$, where μ_0 is the permeability of the free space. The value of μ_0 is

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/amp^2}$$

The expression for field at a point P having a position vector $\vec{r}$ with respect to the current element is

$$d\vec{B} = \frac{\mu_0}{4\pi} I\frac{\vec{dl}\times\hat{r}}{r^2}$$

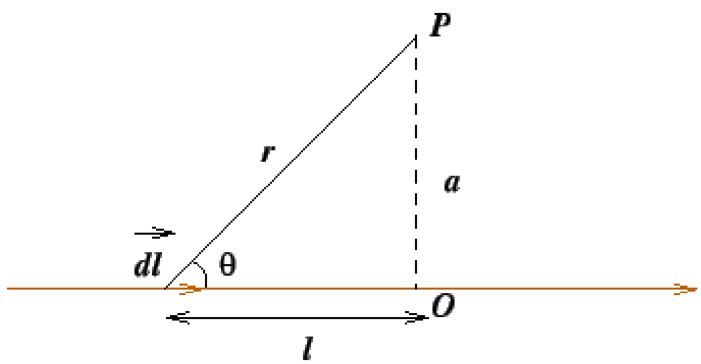


For a conducting wire of arbitrary shape, the field is obtained by vectorially adding the contributions due to such current elements as per superposition principle, $\vec{B}(P) = \frac{\mu_0}{4\pi} I \int \frac{\vec{dl} \times \hat{r}}{r^2}$ where the integration is along the path of the current flow.

Example 5

Field due to a straight wire carrying current

The direction of the field at P due to a current element $\vec{dl}$ is along $\vec{dl} \times \vec{r}$, which is a vector normal to the page (figure on the left) and coming out of it.



We have,

$$\frac{\vec{dl} \times \hat{r}}{r^2} = \frac{|\, dl \sin\theta \,|}{r^2} \hat{k}$$

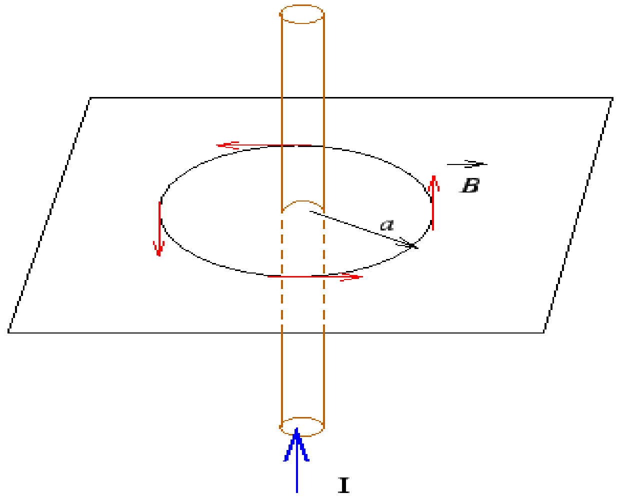
where the plane of the figure is taken as the x-y plane and the direction of outward normal is parallel to z-axis. If a be the distance of the oint P from the wire,

we have
$$\begin{aligned} r &= a/\sin\theta \\ l &= a\cot\theta \\ dl &= -(a/\sin^2\theta)d\theta \end{aligned}$$

Thus

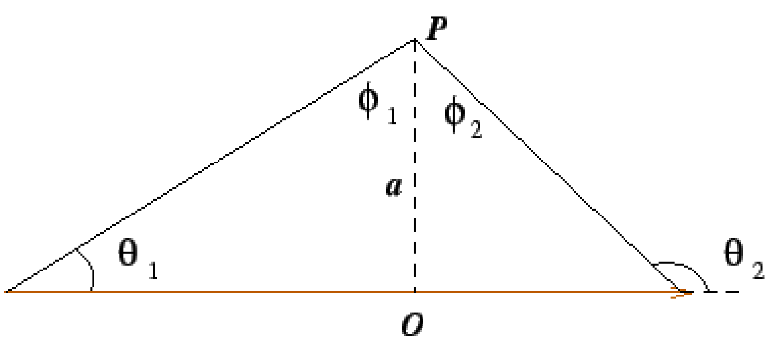
$$\frac{\vec{dl} \times \hat{r}}{r^2} = \frac{\sin\theta}{a} d\theta \hat{k}$$

The direction of the magnetic field at a distance a from the wire is tangential to a circle of radius a, as shown.



Since the magnetic field due to all current elements at P are parallel to the z-direction, the field at P due to a wire, the ends of which make angles ϕ_1 and ϕ_2 at P is given by a straightforward integration

$$\begin{aligned}
\vec{B} &= \frac{\mu_0 I}{4\pi a} \int_{\theta_1}^{\theta_2} \sin\theta d\theta \hat{k} \\
&= \frac{\mu_0 I}{4\pi a} \left[-\cos\theta \right]_{\theta_1}^{\theta_2} \hat{k} \\
&= \frac{\mu_0 I}{4\pi a} \left[-\cos\theta_1 - \cos\theta_2 \right] \hat{k} \\
&= \frac{\mu_0 I}{4\pi a} \left[\sin\phi_1 + \sin\phi_2 \right] \hat{k}
\end{aligned}$$



Note that both the angles ϕ_1 and ϕ_2 are acute angles.

If we consider an infinite wire (also called long straight wire), we have $\phi_1 = \phi_2 = \pi/2$, so that the field due to such a wire is

$$\vec{B} = \frac{\mu_0 I}{2\pi a} \hat{k}$$

where the direction of the field is given by the Right hand rule.

Exercise 1

A conductor in the shape of an n-sided polygon of side a carries current I. Calculate the magnitude of the magnetic field at the centre of the polygon.

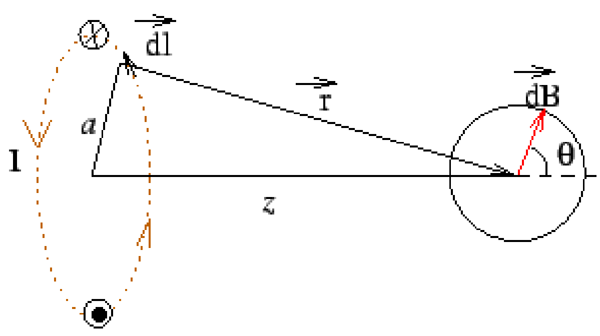
[Ans. $(\mu_0 In/\pi a)\sin(\pi/n)$.]

Example 6

Field due to a circular coil on its axis
Consider the current loop to be in the x-y plane, which is taken perpendicular to the plane of the paper in which the axis to the loop (z-axis) lies. Since all length elements on the circumference of the ring are perpendicular to $\vec{r}$, the magnitude of the field at a point P is given by

$$dB = \frac{\mu_0 I}{4\pi}\frac{dl}{r^2}$$

The direction of the field due to every element is in the plane of the paper and perpendicular to $\vec{r}$, as shown. Corresponding to every element $d\vec{l}$ on the circumference of the circle, there is a diametrically opposite element which gives a magnetic field $d\vec{B}$ in a direction such that the component of $d\vec{B}$ perpendicular to the axis cancel out in pairs.



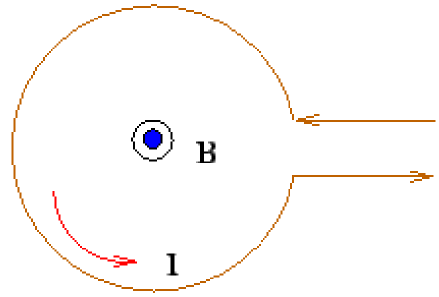
The resultant field is parallel to the axis, its direction being along the positive z-axis for the current direction shown in the figure. The net field is

$$\begin{aligned}\vec{B} &= \frac{\mu_0 I}{4\pi}\int\frac{dl}{r^2}\cos\theta \\ &= \frac{\mu_0 I}{4\pi}\frac{\cos\theta}{r^2}\int dl \\ &= \frac{\mu_0 I\cos\theta}{4\pi r^2}2\pi a = \frac{\mu_0 I a\cos\theta}{2r^2}\end{aligned}$$

In terms of the distance z of the point P and the radius a, we have

$$B = \frac{\mu_0 I}{2}\frac{a^2}{(a^2+z^2)^{3/2}}$$

The direction of the magnetic field is determined by the following Right Hand Rule.



If the palm of the right hand is curled in the direction of the current, the direction in which the thumb points gives the direction of the magnetic field at the centre of the loop. The field is, therefore, outward in the figure shown.

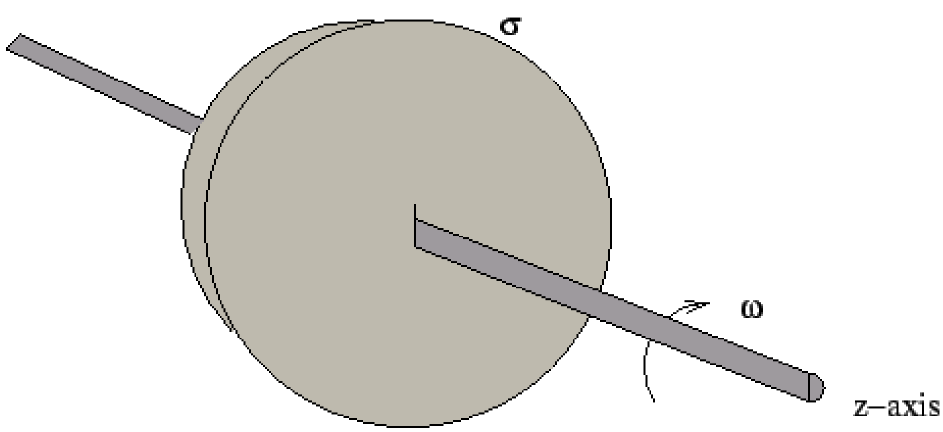
Note that for $z \gg a$, i.e. the field due to circular loop at large distances is given by

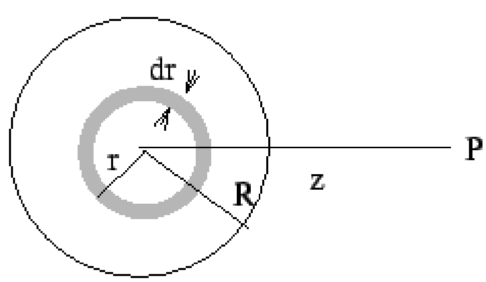
$$B = \frac{\mu_0 I a^2}{2z^3} = \frac{\mu_0 \mu}{2\pi z^3}$$

where $\mu = I\pi a^2$ is the magnetic moment of the loop. The formula is very similar to the field of an electric dipole. Thus a current loop behaves like a magnetic dipole.

Example 7

A thin plastic disk of radius R has a uniform surface charge density σ. The disk is rotating about its own axis with an angular velocity ω. Find the field at a distance z along the axis from the centre of the disk.





The current on the disk can be calculated by assuming the rotating disk to be equivalent to a collection of concentric current loops. Consider a ring of radius r and of width dr. As the disk is rotating with an angular speed ω, the rotating charge on the ring essentially behaves like a current loop carrying current $\sigma \cdot 2\pi r dr \cdot \omega/2\pi = \sigma\omega r dr$.

The field at a distance z due to this ring is

$$dB = \frac{\mu_0(\sigma\omega r dr)}{2}\frac{r^2}{(r^2+z^2)^{3/2}}$$

The net field is obtained by integrating the above from $r = 0$ to $r = R$

$$\begin{aligned} B &= \frac{\mu_0\sigma\omega}{2}\int_0^R \frac{r^3}{(r^2+z^2)^{3/2}}dr \\ &= \frac{\mu_0\sigma\omega}{2}\int_0^R \frac{r^2+z^2-z^2}{(r^2+z^2)^{3/2}} r dr \end{aligned}$$

The integral above may easily be evaluated by a substitution $x = r^2 + z^2$. The result is

$$B = \mu_0\sigma\omega\left[\frac{R^2+2z^2}{(R^2+z^2)^{3/2}} - 2z\right]$$

The field at the centre of the disk ($z = 0$) is

$$B(z=0) = \mu_0\sigma\omega R$$

Exercise 2

Find the magnetic moment of the rotating disk of Example 7.

[Ans. $\pi\omega R^4/4$]

Example 8

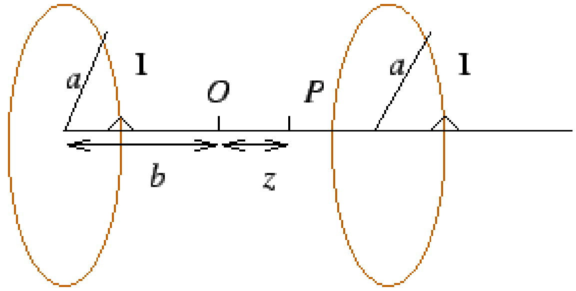
Two coaxial circular coils of radius a each carry current I each in the same sense. The centres of the coils are separated by a distance $2b$. Determine the field along the axis. The set up is called ``Helmholtz coil" when the distance $2b$ between the centres of the coils equals the radius a of each of the loop. The field in the region between the coils of such a coil is nearly uniform.

If the distance z along the axis is measured from the mid points of the line joining the centres of the two coils, the field strength due to the left coil at P is

$$B_1 = \frac{\mu_0 I}{2}\frac{a^2}{[a^2+(b+z)^2]^{3/2}}$$

and that due to the right coil is

$$B_2 = \frac{\mu_0 I}{2}\frac{a^2}{[a^2+(b-z)^2]^{3/2}}$$



The net field at P, due to both coils add up and is given by $B(z)$

$$
\begin{aligned}
&= \frac{\mu_0 I a^2}{2}\left[\frac{1}{[a^2+(b+z)^2]^{3/2}}+\frac{1}{[a^2+(b-z)^2]^{3/2}}\right] \\
&= \frac{\mu_0 I a^2}{2(a^2+b^2)^{3/2}}\left[\frac{1}{[1+\frac{z^2+2zb}{a^2+b^2}]^{3/2}}+\frac{1}{[1+\frac{z^2-2zb}{a^2+b^2}]^{3/2}}\right]
\end{aligned}
$$

We can express the above in a power series using a binomial expansion. Up to z^4, the terms in the expansion may be written as

$$
\left[\frac{1}{[1+\frac{z^2+2zb}{a^2+b^2}]^{3/2}}\right] = 1-\frac{3}{2}\left(\frac{z^2+2zb}{(a^2+b^2)}\right)+\frac{15}{8}\left(\frac{z^2+2zb}{(a^2+b^2)}\right)^2-\frac{35}{16}\left(\frac{z^2+2zb}{(a^2+b^2)}\right)^3+\frac{315}{128}\left(\frac{z^2+2zb}{(a^2+b^2)}\right)^4
$$

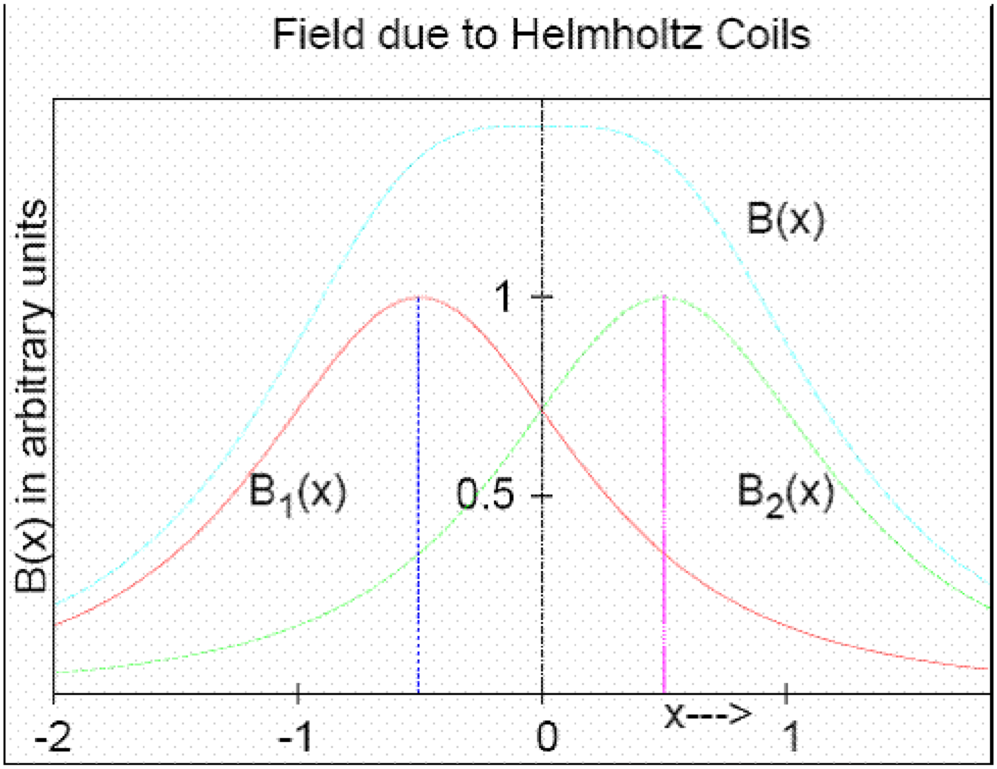
and a similar expression for the second term with $-2zb$replacing $2zb$. Adding the terms and retaining only up to powers of z^4we get

$$
B(z) = \frac{\mu_0 I a^2}{(a^2+b^2)^{3/2}}\left[1-\frac{3}{2}\frac{a^2-4b^2}{(a^2+b^2)^2}z^2+\frac{15}{8}\frac{a^4+8b^4-12a^2b^2}{(a^2+b^2)^4}z^4\right]
$$

For the case of a Helmholtz coil, $2b = a$, and the expression for field is independent of $z$up to its third power, and is given by,

$$
B(z) = \frac{\mu_0 I}{a}(4/5)^{3/2}\left[1-\frac{144}{125}\frac{z^4}{a^4}\right]
$$

It can be seen that the field along the axis is nearly uniform in the region between the coils.



Example 9

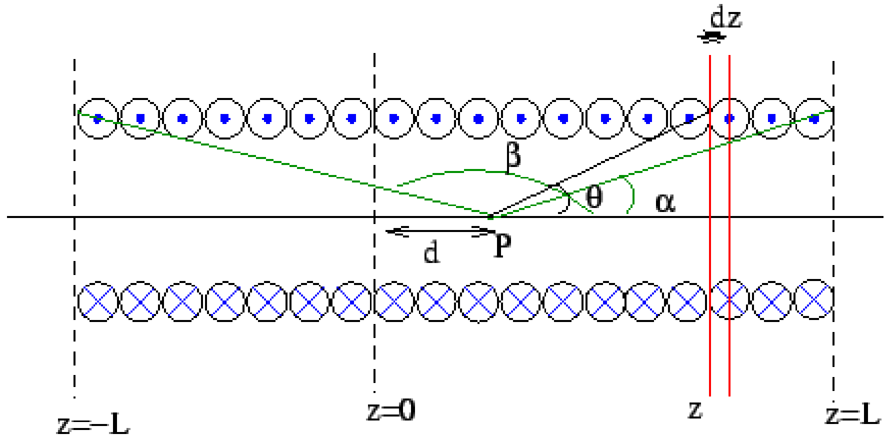
Consider a solenoid of N turns. The solenoid can be considered as stacked up circular coils. The field on the axis of the solenoid can be found by superposition of fields due to all circular coils. Consider the field at P due to the circular turns between z and $z+dz$ from the origin, which is taken at the centre of the solenoid. The point P is at $z=d$. If L is the length of the solenoid, the number of turns within z and $z+dz$ is $Ndz/L = ndz$, where n is the number of turns per unit length.

The magnitude of the field at P due to these turns is given by

$$dB = \frac{\mu_0 N I dz}{2L} \frac{a^2}{[a^2 + (z-d)^2]^{3/2}}$$

The field due to each turn is along $\hat{k}$; hence the fields due to all turns simply add up. The net field is

$$\vec{B} = \frac{\mu_0 N I a^2}{2L} \int_{-L/2}^{L/2} \frac{dz}{[a^2 + (z-d)^2]^{3/2}} \hat{k}$$



The integral above is easily evaluated by substituting

$$\begin{aligned} z - d &= a\cot\theta \\ dz &= -a\mathrm{cosec}^2\theta d\theta \end{aligned}$$

The limits of integration on θ are α and β as shown in the figure. With the above substitution

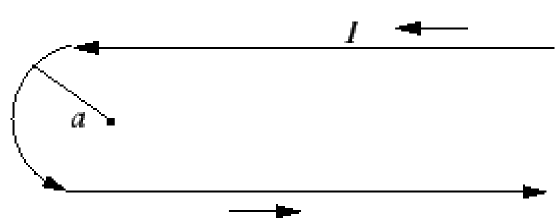
$$\vec{B} = -\frac{\mu_0 nI}{2}\int_\alpha^\beta \sin\theta d\theta = \frac{\mu_0 nI}{2}(\cos\alpha - \cos\beta)\hat{k}$$

For a long solenoid, the field on the axis at points far removed from the ends of the solenoid may be obtained by substituting $\alpha = 0^\circ$ and $\beta = 180^\circ$, so that, $\vec{B} = \mu_0 nI\hat{k}$

The field is very nearly constant. For points on the axis far removed from the ends but outside the solenoid, $\alpha \approx \beta$ so that the field is nearly zero.

Example 10

Determine the field at the point located at the centre P of the semi-circular section of the hairpin bend shown in the figure.



Solution :
The field at P may be determined by superposition of fields due to the two straight line sections and the semicircular arc. The contribution due to all three sections add up as the field due to each is into the plane of the paper.

The field due to each straight line section is obtained by putting $\phi_1 = 90^\circ$ and $\phi_2 = 0^\circ$ in the expression obtained in Example 5 above. The field due to each wire is $\mu_0 I/4\pi a$.

For the semi-circular arc, each length element on the circumference is perpendicular to $\vec{r}$, the vector from the length element to the point P. Thus

$$\begin{aligned} B_{arc} &= \frac{\mu_0 I}{4\pi}\int \frac{\vec{dl}\times\hat{r}}{r^2} \\ &= \frac{\mu_0 I}{4\pi a^2}\int dl = \frac{\mu_0 I}{4\pi a^2}\cdot \pi a = \frac{\mu_0 I}{4a} \end{aligned}$$

The net field due to the current in the hairpin bend at P is

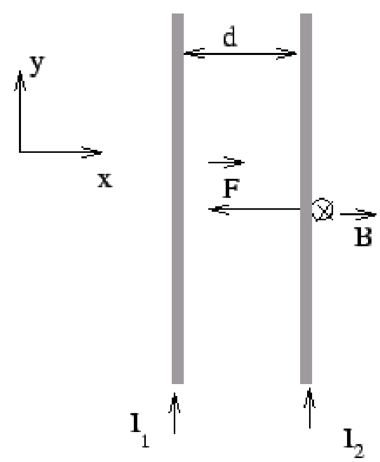
$$B = \frac{\mu_0 I}{2\pi a} + \frac{\mu_0 I}{4a}$$

Example 11

Force due to the first wire at the position of the second wire is given by

$\vec{B} = -\frac{\mu_0 I_1}{2\pi d}\hat{k}$ where $\hat{k}$ is a unit vecor out of the page. The force experienced by the second wire in this field is

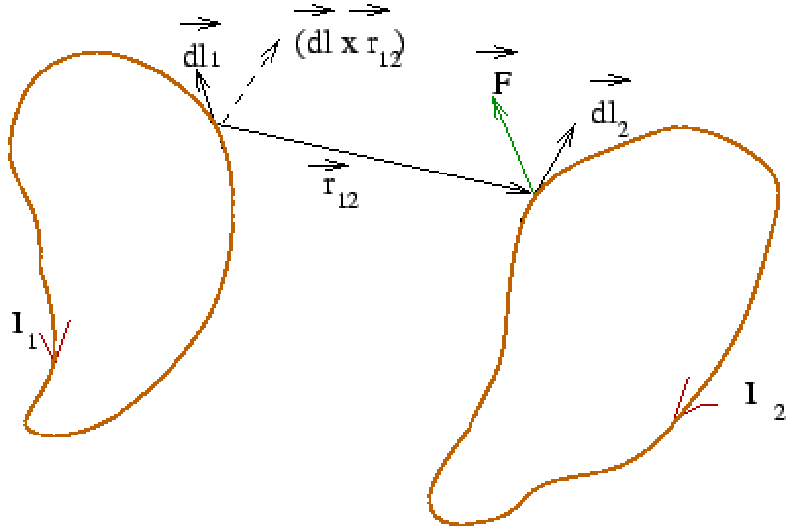
$$\begin{aligned} \vec{F} &= \int (\vec{I_2}\times\vec{B})dl \\ &= -\frac{\mu_0 I_1 I_2}{2\pi d}(\hat{\jmath}\times\hat{k})\int dl \\ &= -\frac{\mu_0 I_1 I_2}{2\pi d}\hat{\imath}\int dl \end{aligned}$$



Thus the force between the wires carrying current in the same direction is attractive and is $\mu_0 I_1 I_2/2\pi d$ per unit length. A generalization of the above is given by the mathematical expression for the force between two arbitrary current loops.

$$\vec{F} = \frac{\mu_0}{4\pi} I_1 I_2 \oint\oint \frac{\vec{dl_2}\times(\vec{dl_1}\times\vec{r_{12}})}{r_{12}^3}$$

where $\vec{r_{12}}$ is the position vector of the element $\vec{dl_2}$ with respect to $\vec{dl_1}$.



Once the integration is carried out, the expression above can be shown to be symmetrical between the two circuits. To show, we express the vector triple product

$$\vec{dl_2} \times (\vec{dl_1} \times \vec{r_{12}}) = \vec{dl_1}(\vec{dl_2} \cdot \vec{r_{12}}) - \vec{r_{12}}(\vec{dl_1} \cdot \vec{dl_2})$$

so that

$$\vec{F} = \frac{\mu_0}{4\pi} I_1 I_2 \left[\oint \oint \frac{\vec{dl_1}(\vec{dl_2} \cdot \vec{r_{12}})}{r_{12}^3} - \frac{\vec{r_{12}}(\vec{dl_1} \cdot \vec{dl_2})}{r_{12}^3} \right]$$

The integrand in the first integral is an exact differential with respect to the integral over $\vec{dl_2}$ as

$$\oint \oint \frac{\vec{dl_1}(\vec{dl_2} \cdot \vec{r_{12}})}{r_{12}^3} = \oint \vec{dl_1} \oint \nabla(\frac{1}{r_{12}}) \cdot \vec{dl_2}$$

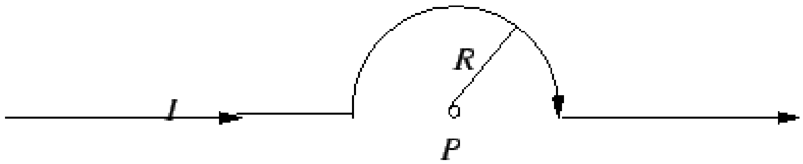
The integral above being an integral of a gradient over a closed path vanishes. Thus

$$\vec{F} = \frac{\mu_0}{4\pi} I_1 I_2 \oint \oint \frac{\vec{r_{12}}(\vec{dl_1} \cdot \vec{dl_2})}{r_{12}^3}$$

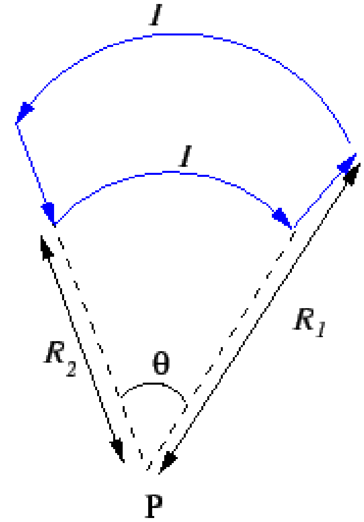
which is explicitly symmetric between the two circuits, confirming the validity of Newton 's third law.

Exercise 3

Determine the magnetic field at the point P for the two geometries shown in the figures below.



(a)



(b)

[Ans . (a) $\mu_0 I/4R$ (b) $\frac{\mu_0 I(R_1 - R_2)\theta}{4\pi R_1 R_2}$]

Recap

In this lecture you have learnt the following

- The magnetic field due to a current element is determined by Biot-Savart's law.
- The magnetic field due to some configurations like a line segment, a circular coil, a disk etc. was calculated using Biot-savart's law.
- A Helmholtz coil is used to produce a uniform magnetic field over a limited region of space.
- A force is exerted on a current element placed in a magnetic field.
- Two current carrying circuits exert force on each other because the magnetic field due to one circuit exerts force on the current elements of the other circuit.

Module 3 : MAGNETIC FIELD
Lecture 16 : Ampere's Law

Objectives

In this lecture you will learn the following

- Establish Ampere' law in integral form.
- Calculate the magnetic field for certain current configuration using Ampere's law.
- Derive the differential form of Ampere's law.

Ampere's Law

Biot-Savart's law for magnetic field due to a current element is difficult to visualize physically as such elements cannot be isolated from the circuit which they are part of. Andre Ampere formulated a law based on Oersted's as well as his own experimental studies. Ampere's law states that `` *the line integral of magnetic field around any closed path equals* μ_0 *times the current which threads the surface bounded by such closed path.* . Mathematically,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enclosed} \qquad (1)$$

In spite of its apparent simplicity, Ampere's law can be used to calculate magnetic field of a current distribution in cases where a lot of information exists on the behaviour of $\vec{B}$. The field must have enough symmetry in space so as to enable us to express the left hand side of (1) in a functional form. The simplest application of Ampere' s law consists of applying the law to the case of an infinitely long straight and thin wire.

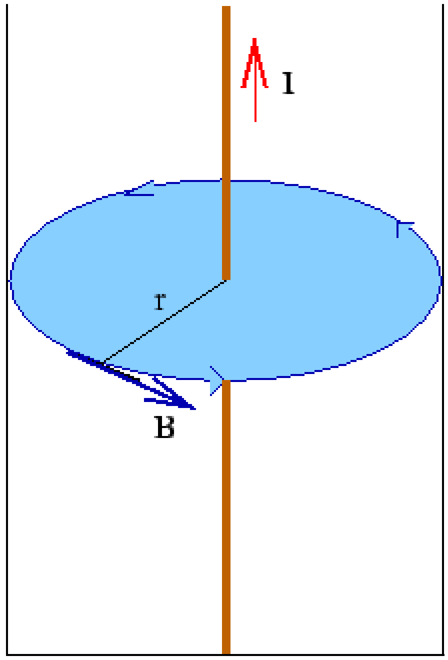
Example 12

By symmetry of the problem we know that the magnitude of the field at a point can depend only on the distance of the point from the wire. Further, the field is tangential to the circle of radius r, its direction being given by the right hand rule.

Thus the integral around the circle is $\oint \vec{B} \cdot \vec{dl} = B \oint dl = B \cdot 2\pi r$

Equating this to $\mu_0 I$, we get $B = \dfrac{\mu_0 I}{2\pi r}$

which is consistent with the result obtained from Biot-Savart's law.



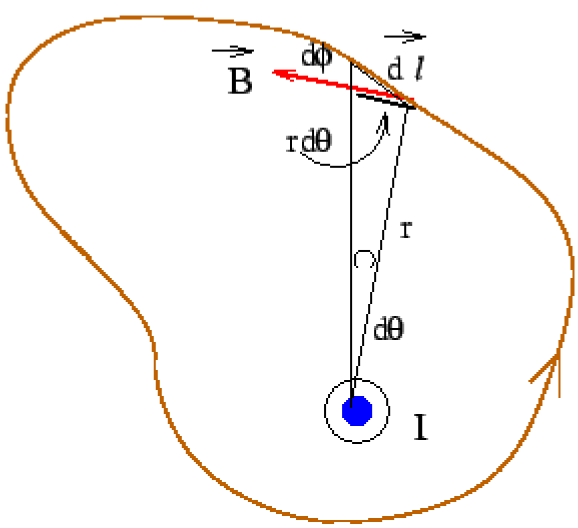
Let us see if the result above is consistent with a path which is not circular, as shown in the figure. The field at every element $\vec{dl}$ of the path is perpendicular to $\vec{r}$. From geometry, it can be seen that

$$\vec{B} \cdot \vec{dl} = B dl \cos\phi = B r d\theta$$

Thus

$$\oint \vec{B} \cdot \vec{dl} = \oint \frac{\mu_0 I}{2\pi r} r d\theta$$
$$= \frac{\mu_0 I}{2\pi} \oint d\theta = \mu_0 I$$

We need to specify the direction along which the path is traversed. This is done by Right Hand Rule. If we curl the fingers of our right hand along the path of integration, the direction along which the thumb points is the direction of current flow.

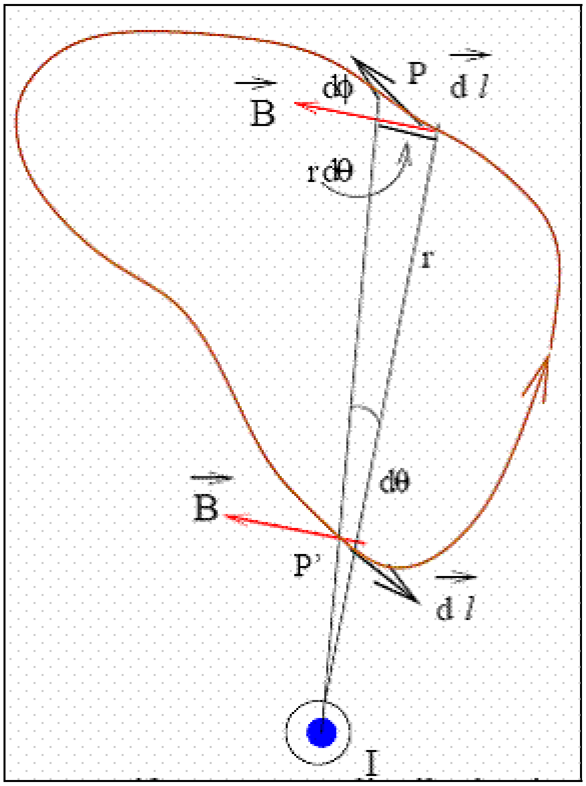


For the case where the path of integration lies totally outside the path of the current, for every element $\vec{dl}$ at

P, there exists another element at P' for which $\vec{B}\cdot\vec{dl}$ has opposite sign. Thus when complete line integral is taken, the contributions from such pairs add to zero

$$\oint \vec{B}\cdot\vec{dl} = 0$$

Combining these, we get Ampere's law in the form of Eqn. (1)



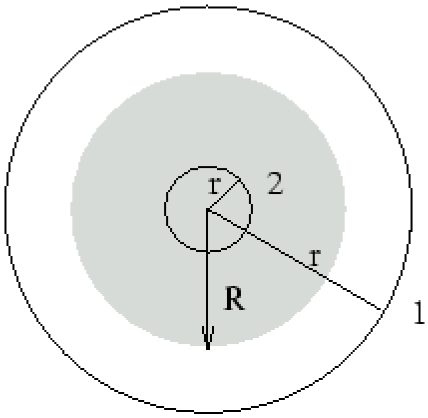
Example 13

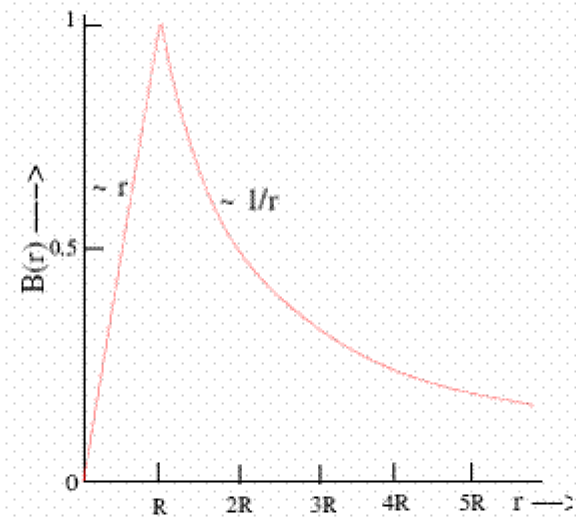
Calculate the field due to a uniform current distribution in an infinite wire of cross sectional radius R.

Solution :

Let the cross section of the wire be circular with a radius R. Take the current direction to be perpendicular to the page and coming out of it. Symmetry of the problem demands that the magnitude of the field at a point is dependent only on the distance of the point from the axis of the wire. Consider an amperian loop of radius r. As before we have

$$\oint \vec{B}\cdot\vec{dl} = 2\pi r B$$





If $r > R$(as in loop 1), the entire current is enclosed by the loop. Hence $I_{enclosed} = I$ so that

$$B = \frac{\mu_0 I}{2\pi r}$$

If $r < R$(loop 2), the current enclosed is proportional to the area, i.e.

$$I_{enclosed} = I\frac{\pi r^2}{\pi R^2} = I\frac{r^2}{R^2}$$

so that

$$B = \frac{\mu_o I}{2\pi R^2} r$$

The field distribution with distance is as shown.

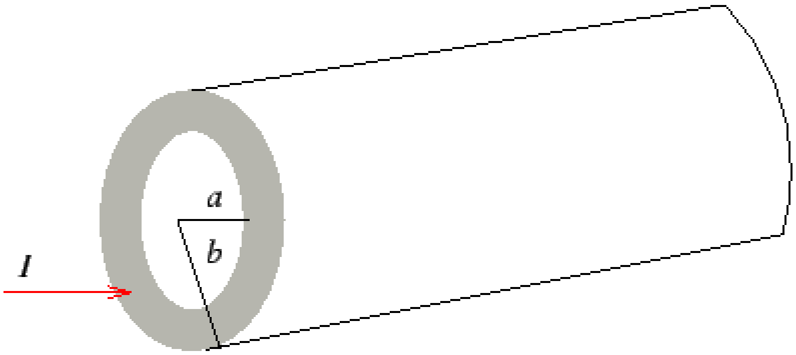
Exercise 1

A long wire of cross sectional radius Rcarries a current I. The current density varies as the square of the distance from the axis of the wire. Find the magnetic field for $r < R$and for $r > R$.

(Hint : First show that the current density $J = 2Ir^2/\pi R^4$and obtain an expression for current enclosed for $r < R$. Answer : $B = \mu_0 I/2\pi r$ for $r > R$and $B = \mu_0 I r^3/2\pi R^4$for $r < R$.)

Exercise 2

A hollow cylindrical conductor of infinite length carries uniformly distributed current I from $a < r < b$. Determine magnetic field for all r.

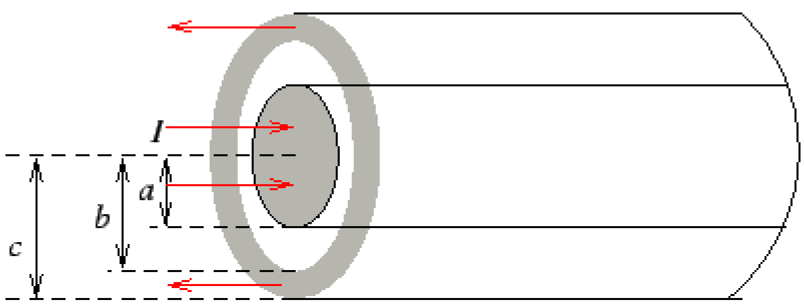


(Answer : Field is zero for $r < a$, $B = \dfrac{\mu_0 I}{2\pi r}\dfrac{r^2 - a^2}{b^2 - a^2}$ for $a < r < b$ and $B = \mu_0 I/2\pi r$ for $r > b$

.)

Exercise 3

A coaxial cable consists of a solid conductor of radius a with a concentric shell of inner radius b and outer radius c. The space between the solid conductor and the shell is supported by an insulating material.



A current I goes into the inner conductor and is returned by the outer shell. Assume the cuurent densities to be uniform both in the shell and in the inner conductor. Calculate magnetic field everywhere.

(Ans. $B = \mu_0 I r/2\pi b^2$ inside the inner conductor, $B = \mu_0 I/2\pi r$ between the shell and the inner conductor, $B = \dfrac{\mu_0 I}{2\pi r}\dfrac{c^2 - r^2}{c^2 - b^2}$)

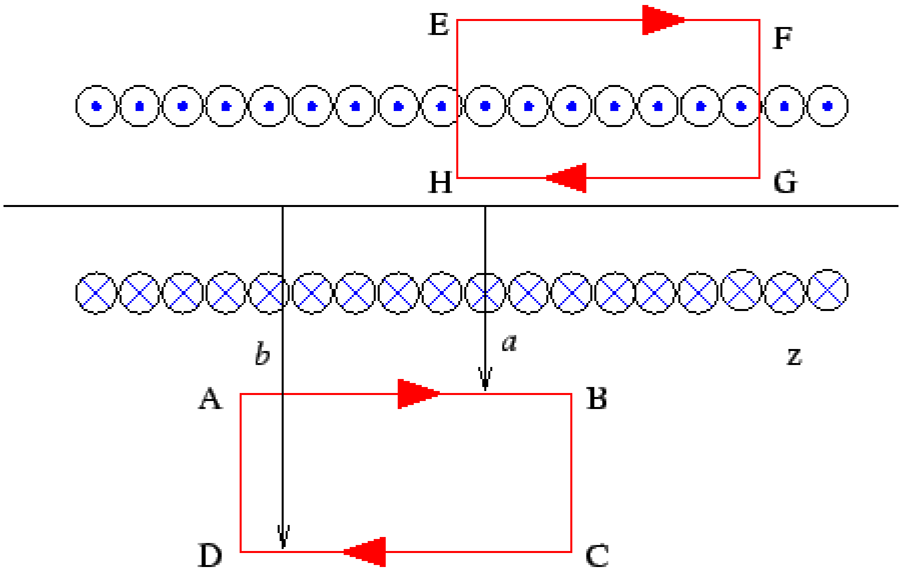
Exercise 4

Determine the magnetic field in a cylindrical hole of radius a inside a cylindrical conductor of radius b. The cylinders are of infinite length and their axes are parallel, being separated by a distance d. The conductor carries a current I of uniform density.

(Hint : The problem is conveniently solved by imagining currents of equal and opposite densities flowing in the hole and using superposition principle to calculate the field. Answer : The field inside the hole is constant $B = \mu_0 I d/2\pi(b^2 - a^2)$)

Example 14

We take the solenoid to be closely wound so that each turn can be considered to be circular. We can prove that the field due to such a solenoid is entirely confined to its interior, i.e. the field outside is zero, To see this consider a rectangular amperian loop parallel to the axis of the solenoid.



Field everywhere on AB is constant and is $B(a)$. Likewise the field everywhere on CD is $B(b)$. By Right hand rule, the field on AB is directed along the loop while that on CD is oppositely directed. On the sides AD and BC, the magnetic field direction is perpendicular to the length element and hence $\vec{B}\cdot\vec{dl}$ is zero everywhere on these two sides. Thus

$$\oint \vec{B}\cdot\vec{dl} = L(B(a) - B(b)$$

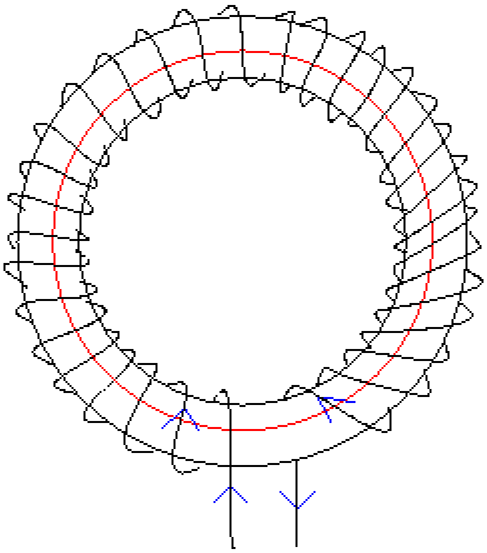
By Ampere's law, the value of the integral is zero as no current is enclosed by the loop. Thus $B(a) = B(b)$. The field outside the solenoid is, therefore, independent of the distance from the axis of the solenoid. However, from physical point of view, we expect the field to vanish at large distances. Thus $B(a) = B(b) = 0$.

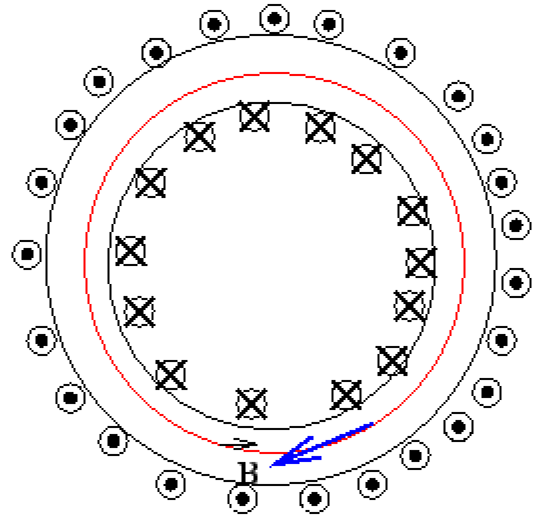
To find the field inside, take an amperian loop EFGH with its length parallel to the axis as before, but with one of the sides inside the solenoid while the other is outside. The only contribution to $\oint \vec{B}\cdot\vec{dl}$ comes from the side GH. Thus, $\oint \vec{B}\cdot\vec{dl} = BL = \mu_0 I_{enclosed} = \mu_0 nLI$

where I is the current through each turn and n is number of turns per unit length. $I_{enclosed} = nLI$ because the number of turns threading the loop is nL. Hence, $B = \mu_0 nI$ is independent of the distance from the axis.

Exercise 5

A toroid is essentially a hollow tube bent in the form of a circle. Current carrying coils are wound over it. Use an amperian loop shown in the figure to show that the field within the toroid is $\mu_0 NI/L$, where N is the number of turns and L the circumference of the circular path.



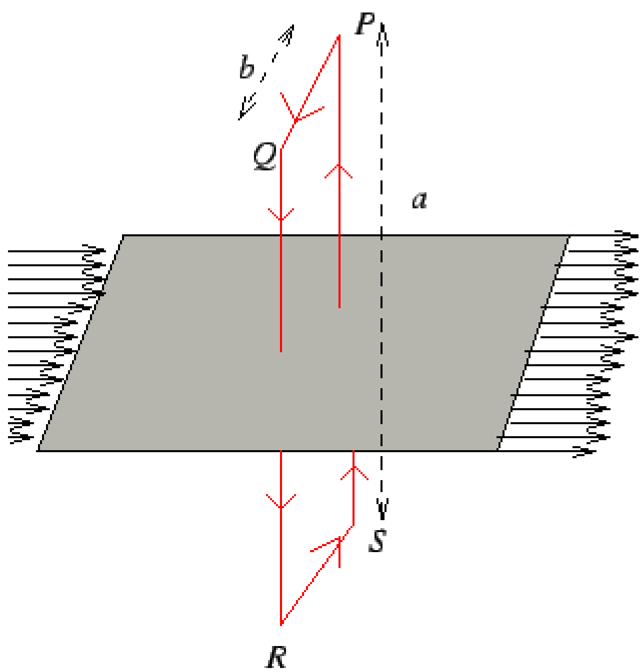


Note that as the circumference of the circular path varies with the distance of the amperian loop from the toroid axis, the magnetic field in the toroid varies over its cross section. Take the inner radius of the toroid to be 20cm and the outer radius as 21cm. Find the percentage variation of the field over the cross section of the toroid.

(Ans. 2.9%)

Example 15

An infinite conducting sheet carries a current such that the current density is λ per unit length. Take an amperian loop as shown.



The contribution to the line integral of $\vec{B}$ from the sides QR and SP are zero as $\vec{B}$ is perpendicular to $\vec{dl}$. For PQ and RS the direction of $\vec{B}$ is parallel to the path. Hence $\oint \vec{B} \cdot \vec{dl} = 2bB = \mu_0 \lambda b$ giving

$B = \mu_0 \lambda / 2$.

Exercise 6

Calculate the force per unit area between two parallel infinite current sheets with current densities λ_1 and λ_2 in the same direction.

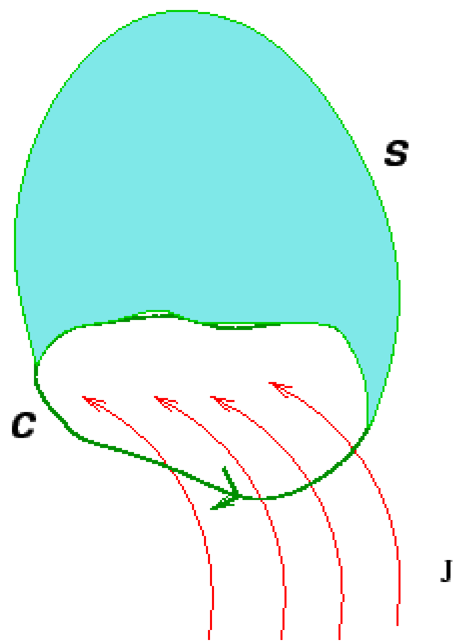
(Ans. $\mu_0\lambda_1\lambda_2/2$)

Ampere's Law in Differential Form

We may express Ampere's law in a differential form by use of Stoke's theorem, according to which the line integral of a vector field is equal to the surface integral of the curl of the field,

$$\oint \vec{B}\cdot\vec{dl} = \int_S \text{curl } \vec{B}\cdot\vec{dS}$$

The surface S is *any* surface whose boundary is the closed path of integration of the line integral.



In terms of the current density $\vec{J}$, we have, $\int_S \vec{J}\cdot\vec{dS} = I_{encl}$

where I_{encl} is the total current through the surface S. Thus, Ampere's law $\oint \vec{B}\cdot\vec{dl} = I_{encl}$ is equivalent to

$$\int \text{curl } \vec{B}\cdot\vec{dS} = \mu_0\int \vec{J}\cdot\vec{dS}$$

which gives $\text{curl } \vec{B} = \mu_0\vec{J}$

You may recall that in the case of electric field, we had shown that the divergence of the field to be given by $\nabla\vec{E} = \rho/epsilon_0$. In the case of magnetic field there are no free sources (monopoles). As a result the divergence of the magnetic field is zero

$$\nabla \cdot \vec{B} = 0$$

The integral form of above is obtained by application of the divergence theorem

$$\int_S \vec{B} \cdot d\vec{S} = \int_V \nabla \cdot \vec{B} dV = 0$$

Thus the flux of the magnetic field through a closed surface is zero.

Recap

In this lecture you have learnt the following

- Ampere's law was stated in integral form and used to calculate magnetic field in symmetric situations.
- Calculation of magnetic field was done due to a a long straight wire, a coaxial cable, a solenoid, a toroid and a current sheet done using Ampere's law.
- Using Stoke's theorem Ampere's law can be expressed in a differential form.